[12] K. Kurokawa, "Some basic characteristics of broadband negative resistance oscillator circuits," *Bell Syst. Tech. J.*, vol. 48, pp. 1937–1955, July–Aug. 1969.

[13] C. G. Montgomery, R. H. Dicke, and E. M. Purcell, Eds., *Principles of Microwave Circuits.* New York, NY: McGraw-Hill, 1948, ch. 7, pp. 230–231.

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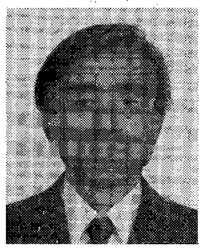

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Short Papers

Coaxial E-Field Probe for High-Power Microwave Measurement

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***Abstract*—Open-ended semirigid coaxial cable is characterized for use as a coupling element for the measurement of high-power microwaves. There are many high-power microwave experiments where a high vacuum must be maintained, and yet it is necessary to measure the power and frequency of the microwave energy in a cavity or waveguide. Semirigid coaxial cable which is left open-ended and inserted no more than flush with the inner waveguide wall is a convenient way to couple out a known sample of the power. The cable is inserted into the waveguide in a direction parallel to the electric field. The coupling value is expressed in terms of an area multiplier which is applied to the area of the end of the center conductor. The induced charge Q on the center conductor is then determined from $Q = \vec{D} \cdot \hat{n} \cdot$(effective area), and the coupled power is calculated from $P = (\omega Q)^2/(2Z_0)$. For a flush mounted $Z_0 = 50\text{-}\Omega$ coax with a PTFE dielectric, the area multiplier is shown to be 3.846 theoretically and 3.77 experimentally. The area multiplier is also determined for various withdrawal depths of the coax into the waveguide wall.**

I. Introduction

In the measurement of microwaves from short-pulse, high-power microwave generators, the usual quantities desired are output power time history, frequency, and the mode structure. These quantities are typically measured with commercially available couplers, filters, and (< 100-mW) detectors where the generator may be producing > 1 gW of peak power in an overmoded circular waveguide. The primary problem is to sample a small fraction of the power and to transmit the power out of the vacuum region using standard waveguide or coaxial cable to a point where the power and frequency can be measured. The best way to do this is with a directional coupler matched to each mode of interest [1]. But couplers from evacuated circular waveguide into rectangular waveguide are expensive and difficult to fabricate. The time required to fabricate the couplers can also discourage their use. A section of open-ended semirigid cable inserted so that it couples out the radial electric field is described here as a convenient and accurate way to measure E_r and, thus, determine the frequency and power of the assumed mode.

This type of probe is commonly known as an "E-field probe" and is being used as a compact sensor in permittivity studies. For those studies, the effect of dielectrics on the reflection coefficient when the open end of a coaxial cable is inserted into a sample has been modeled by Gajda and Stuchly [2], and by Stuchly *et al.* [3]. However, these models consider power flowing from a generator towards the open end where it is reflected back towards the generator. Here, we address the case where power is coupled into the open end and flows to a matched termination. Smith [4] has done some studies with electrically short coaxial monopoles, but they were fixed length probes, protruding above the metal wall and did not use matched terminations. The E probe we discuss here is inserted into the waveguide through a hole (Fig. 1), yet it is not inserted more than flush with the inner waveguide wall. This makes it possible to leave the outer conductor on the cable, avoiding an additional discontinuity. The cable is vacuum sealed

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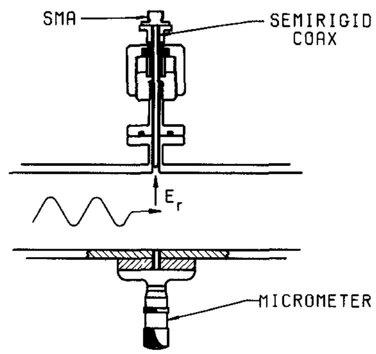

Fig. 1. E-field probe in an adjustable mount is shown mounted on the rectangular waveguide E-probe calibration device. The normal electric field E_r couples into the coaxial probe.

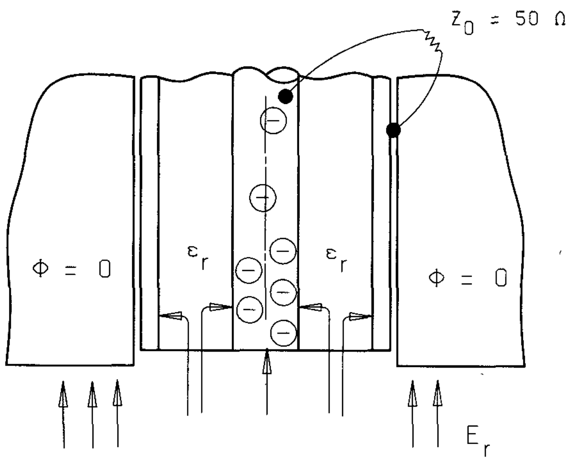

Fig. 2. Close-up drawing of the open-ended semirigid coaxial E probe. The incident electric field E_r induces charge on the center conductor. As E_r varies sinusoidally, the charge must flow through the cable, which is represented here by the characteristic impedance $Z_0 = 50\ \Omega$.

with an "O" ring and nut. Withdrawal relative to the flush setting can be accomplished simply by loosening the nut and moving the probe for a different coupling value. Since only the radial electric field at the wall is measured, assumptions must be made as to what mode is propagating, azimuthal symmetry, and VSWR. Once these assumptions are satisfied, accurate measurements can be made.

II. Theory

The coaxial cable diameter is a small fraction of a wavelength for the frequencies of interest. For example, the first coaxial mode is cutoff at 33 GHz for 0.141-in OD semirigid cable. This allows the coupling to the TEM mode to be analyzed quasistatically. Consider the coaxial cable in Fig. 2, where the normal electric field generates a surface charge density on the center conductor of the coax. At zero frequency, the center conductor is held at zero potential through the 50-Ω termination. When E_r varies, the charge varies linearly, but the current path is through the resistor Z_0 with the dissipated power representing the power that is coupled down the coaxial transmission line. Through quasistatic arguments, the behavior at frequencies well below the 33-GHz cutoff in 0.141 semirigid cable should be well approximated by the zero frequency case.

The inner conductor surface charge is given by

$$Q(t) = KD_r(t) \tag{1}$$

where K is the effective area of the center conductor.

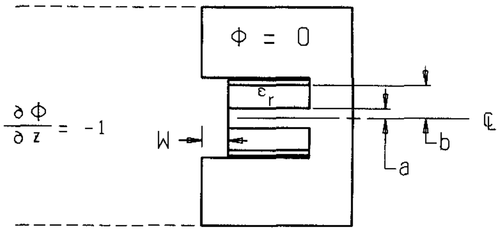

Fig. 3 Computer model for the coaxial E-probe coupling problem with a variable withdrawal depth W. A fixed electric field of 1 V/cm is normal to the left surface. The charge Q was calculated by integrating $D \cdot \hat{n}$ over a Gaussian surface surrounding the center conductor.

Then, for sinusoidal $E_r(t)$, the current flow and coupled power become

$$I(t) = \frac{dQ(t)}{dt} = Kj\omega D_r(t) = Kj\omega\epsilon_0 E_r e^{j\omega t} \tag{2}$$

$$P = \frac{1}{2}|I|^2 Z_0 = \frac{Z_0}{2}[\omega\epsilon_0 E_r K]^2. \tag{3}$$

Thus, if we can determine the constant K, the normal electric field can be derived by simple power measurements.

The constant K was determined by solving Laplace's equation using the computer code JASON [5]. The code solves

$$\nabla \cdot \epsilon \nabla \phi + \rho = 0 \tag{4}$$

where ϵ is the dielectric tensor, diagonal for this case, ϕ is the electrostatic potential, ρ is the free-charge density, and, for our use, we set $\rho = 0$. The boundary conditions (Fig. 3) are $\phi = 0$ on the right surfaces and $\partial\phi/\partial z = -1$ on the left surface. The boundary conditions on the dotted lines are $E_r = 0$. Variable sized zoning is allowed in the code, so tight zoning was used close to the center conductor to obtain the best accuracy. The center conductor was modeled using 20 zones radially, and 18 zones were used in the dielectric region. The dielectric was set for $\epsilon_r = 2.023$, and the calculations are valid only for 50-Ω coaxial cable.

To remove the dimensions from the constant K and from the withdrawal depth W, we introduce the dimensionless variables

$$K_r(W_r) = K(W_r)/(\pi a^2), \quad \text{area multiplication factor for relative withdrawal } W_r \tag{5}$$

$$W_r = W/b, \quad \text{withdrawal depth as a fraction of the outer radius } b. \tag{6}$$

The results from the Laplace solver JASON are shown in Fig. 4 along with the experimental results.

To use this graph, K_r is read based on the set withdrawal depth as a fraction of b. For example, consider the following case where we couple the radial electric field E_r from a circular waveguide into the E probe:

Circular Waveguide:
Radius = 4.25 cm
Power = 100 MW
Frequency = 7.0 GHz
Mode = TM_{01}

Coaxial Probe:
0.141-in semirigid cable
$a = 0.456$ mm
$b = 1.492$ mm
$W_r = 0.0$

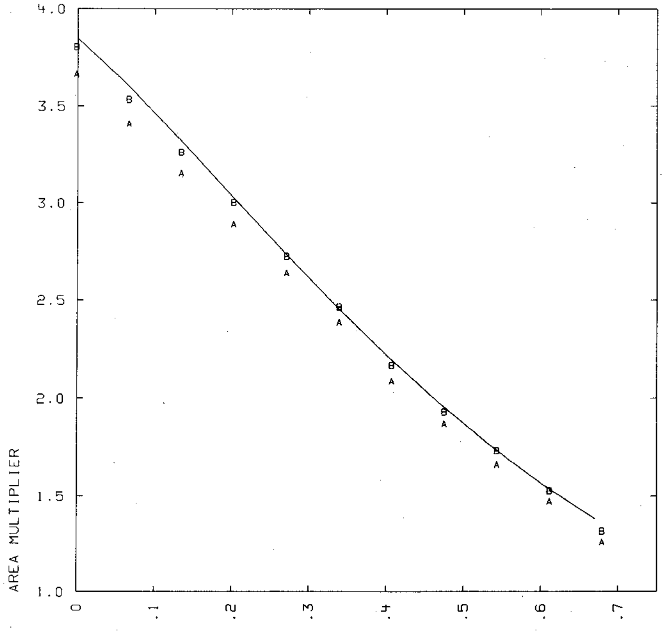

Fig. 4. Area multiplier K_r versus withdrawal for the E probe. Solid line is the theoretical result calculated by the Laplace solver JASON. The measured results for 7.0 GHz are shown as the points A. The measured results for 7.5 and 8.0 GHz were nearly identical and are shown as the points B.

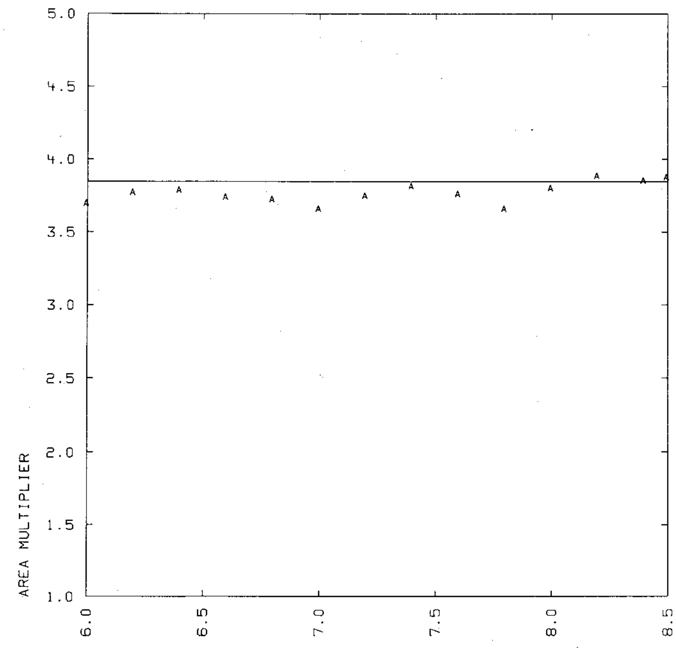

Fig. 5. Coaxial E-probe area multiplier K_r calculated from measured P/E_r for a range of frequencies and zero withdrawal. The straight line is the theoretical $K_r = 3.846$.

From Fig. 4 and (5), we get $K_r = 3.85$ and $K = 2.515 \times 10^{-6}$ m^2. From [6], we calculate the peak radial electric field at the waveguide wall to be 3.50×10^6 V/m after correcting [6, eq. 8.04.11] by a factor of 2. Putting these factors in (3), we get $P = 294$ W, or a coupling value of 55.3 dB.

III. Experimental Results

The electric-field coupling into the open-ended coaxial E probe was measured using the apparatus shown in Fig. 1. A 0.141-in semirigid cable was inserted into the center broadwall of a precision WR-112 waveguide using an adjustable mount. Opposite the cable, a micrometer was fixed so it could measure the probe withdrawal, yet be adjusted flush with the wall during the microwave measurements. To minimize the standing wave, a tuner was inserted prior to the terminator and VSWR was reduced to 1.01 or less during the coupling measurements. For each frequency, the waveguide power and the E-probe power were measured and K_r was determined from (3) and (5) which are rewritten here as

$$K_r = \frac{1}{\pi a^2 \epsilon_0} \sqrt{\frac{2}{Z_0}} \frac{\sqrt{P}}{\omega E_r}. \tag{7}$$

E_r was determined from the frequency and the waveguide power measurement [6]. The results of this measurement are shown in Fig. 5 along with the theoretical constant area multiplier $K_r = 3.846$ for zero withdrawal depth. The periodic fluctuation in the measured K_r corresponds to a discontinuity at the connector which is 4.5 in from the open end of the probe. The average $K_r = 3.77$ agrees well with the theoretical value, and if the calculated attenuation of the cable is taken into account, K_r would be 3.88 experimentally.

As the E probe was withdrawn into its hole, the coupling was measured and plotted in terms of the area multiplier K_r in Fig. 4. The solid line is the theoretical result and the data points for 7.0, 7.5, and 8.0 GHz are plotted on the graph. The 7-GHz results are below the theoretical, but the 7.5- and 8.0-GHz results are nearly identical and follow the theoretical result quite well.

The calculations and measurements are valid for any flush mounted 50-Ω cable with $\epsilon_r = 2.023$. However, as the cable is withdrawn into its hole, the reduction in the coupling is a function of the hole diameter, which would be close to the outer conductor outside diameter. The outer conductor OD is not scaled with the inner conductor for different size cables so that only M17/130-RG402 (0.141 semirigid) and M17/129-RG-401 (0.250 semirigid) are described by this paper. M17/133-RG-405 (0.085 semirigid) will not decrease its coupling as quickly when withdrawn as shown in Fig. 4, although by dividing W_r by 1.1 to 1.2 and using Fig. 4, one should be able to obtain reasonable results for 0.085-, 0.047-, 0.034-, and 0.013-in semirigid cables.

The preparation of the E probe was very critical since an error in end flatness of 0.004 in can cause a 7-percent coupling error. It is very easy to form radial burrs on the end of the inner conductor when machining, which we did with a high-speed lathe. The standard semirigid cable is made with silver plated copper clad steel which will rust when it is left exposed. We used cable with a silver plated Be–Cu core which maintained a nice finish and would machine to a nearly burr-free end. Since the E probe $\Gamma_s \approx 1$, the load match had to be very good to avoid a large ripple, so extra care was taken when mounting connectors to the probe and an attenuator of at least 6 dB was installed as the first element after the connector.

IV. Conclusions

The coupling into a flush-mounted open-ended semirigid coaxial cable was calculated based on a static capacitive model. The coupling was measured using traveling waves in a WR-112 waveguide and a 0.141-in OD semirigid coaxial E-field probe. The coupling value was determined in terms of an area multiplier K_r, which defined the effective area of the inner conductor of the open-ended cable. The induced charge on the inner conductor of radius a is then simply $Q = \epsilon_0 E_r [K_r \pi a^2]$ and coupled power $P = (1/2) Z_0 (\omega Q)^2$, where E_r is the peak normal electric field and $Z_0 = 50\ \Omega$. K_r was calculated using a Laplace solver on a CDC-7600 to be 3.846 and was measured to be 3.77.

The coupling was also calculated and measured as the probe was withdrawn into the waveguide wall to obtain as much as 9-dB less coupling than the flush coupling value.

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References

[1] S. E. Miller, "Coupled wave theory and waveguide applications," *Bell Syst. Tech. J.* pp. 661–719, May 1954.
[2] G. B. Gajda and S S. Stuchly, "Numerical analysis of open-ended coaxial lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-31, pp. 380–384, May 1983.
[3] M. A. Stuchly, M. M. Brady, S. S. Stuchly, and G. B Gajda, "Equivalent circuit of an open-ended coaxial line in a lossy dielectric," *IEEE Trans Instrum. Meas.*, vol. IM-31, pp. 116–119, June 1982
[4] G. S. Smith, "A comparison of electrically short bare and insulated probes for measuring the local radio frequency electric field in biological systems," *IEEE Trans. Bio-Med. Eng.*, vol. BME-22, no. 6, pp. 477–483, Nov. 1975
[5] S. J Sackett, "JASON—A code for solving general electrostatics problems," *Lawrence Livermore National Laboratory Rep.* No. UCID-17814.
[6] S. Ramo, J R. Whinnery, and T. Van Duzer, *Fields and Waves in Communication Electronics.* New York: Wiley, 1965, pp. 420–434

Computation of Inductance of Simple Vias Between Two Striplines Above a Ground Plane

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Abstract —**In this paper, an analysis is developed for calculating the lumped inductance of a simple via connecting two infinitely thin striplines, located above a perfectly conducting ground plane. The striplines are oriented in the same direction, and the via is assumed to be in the form of an infinitely thin vertical plate, connecting the two lines. This system is analyzed by a hybrid partial-element and circuit-theory approach. Numerical results are presented to illustrate the application of this technique.**

I. Introduction

In order to provide an accurate analysis of electronic circuits containing transmission lines, it is necessary to take into account the discontinuities of the transmission lines, such as the line terminations, bends, crossovers, and connections between different transmission lines (vias). In the simplest model, these discontinuities are represented by equivalent lumped-element networks, consisting of inductors and capacitors.

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The lumped elements representing the discontinuities are obtained by static analyses (magnetostatic and electrostatic). These elements, essentially, take into account the excess magnetic and electric energy stored in the field in the vicinity of the discontinuity, as compared to the energy stored in the field along a uniform transmission line. Therefore, it is possible in some cases that the inductances or capacitances have negative values.

In this paper, an analysis is developed for calculating the lumped inductance of a simple via connecting two infinitely thin striplines, located above a perfectly conducting ground plane. The striplines are oriented in the same direction, and the via is assumed to be in the form of an infinitesimally thin vertical plate, connecting the two transmission lines.

Only a few papers exist dealing with the analysis of similar structures. A brief survey of the existing techniques is given in [1]. The so-called partial-element method [2]–[3] is based on dividing the conductor into a number of rectangular elements, and the self and mutual inductances of these elements are evaluated. Resistances can also be included in such a procedure. These inductances (and possible resistances) are thereafter interconnected so to form a network, which is solved by standard circuit-theory techniques. In the second group of papers [4]–[5], the integral equations and the Galerkin's method are used to solve for the unknown current distribution in the conductors. Although at first glance these two methods seem to be different, the partial-element method is, basically, very closely related to Galerkin's technique.

In this paper, we essentially employ the partial-element method, given in [3]. A brief description of the method is given in Section II. It is worth mentioning that the method gives, as a byproduct, the inductances per unit length of the two striplines, joined by the via. In Section III, some numerical examples are given, showing the dependence of the via equivalent inductance on the via dimensions.

II. Basic Principles

Let us consider a perfectly conducting body which has two well-defined terminals (i.e., has a distinct port). Our objective is to find the inductance of this structure as seen from the port. The method which we are going to apply for solving this problem is based on the partial-element method, described in [3]. It is a hybrid of the electromagnetic-field methods and the electric-circuit methods. Although we are not going to explicitly use the frequency-domain analysis, our solution corresponds essentially to the limiting case of time-harmonic fields when the frequency tends to zero. This situation is sometimes referred to as the magnetostatic analysis.

First note that the currents in our object are located only in a surface layer, because the structure is perfectly conducting. The surface-current density vector $\boldsymbol{J}_s$ at any point on the surface can be represented as the sum of two orthogonal components. For the sake of simplicity, we shall assume that the surface of the conductor is piecewise flat and that the current-density vector can be represented in terms of two local, say, u and v, components. We first approximate the conductor by surface patches (partial elements) carrying currents of constant density over a patch that will represent the body regarding the magnetic field it produces, and treat these patches as simple inductances. Next, we create a network of these inductances and thus find the total inductance between the input ports. Enforcing the first Kirchhoff's law for all the nodes of the network ensures that the